Problems

1 The state space diagram and transition rates in f/hr of a continuous Markov process is shown in Figure 9.9. Calculate:
   (a) The limiting probabilities of each state.
   (b) The availability of the system.
   (c) The MTTF for the following system operating conditions;
      (1) state 1 is the normally up state, state 2 is a standby state and state 3 is the failure state; and
      (2) state 1 is the operating state and states 2 and 3 are failure states.

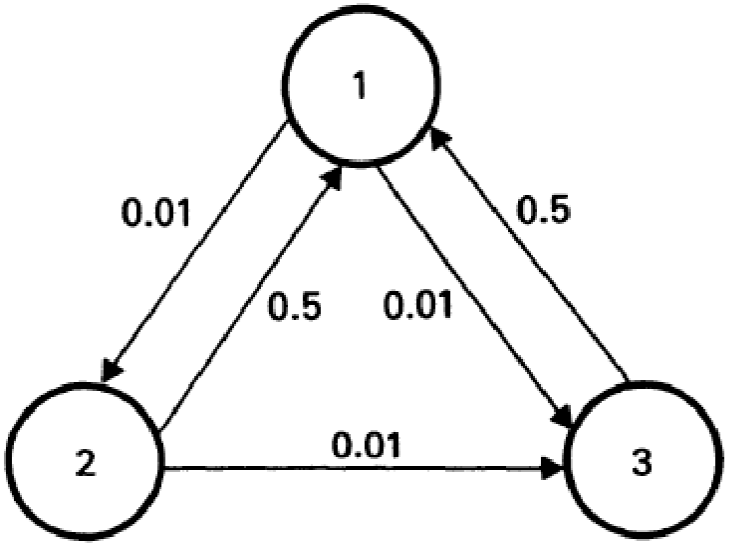

Fig. 9.9

2 The following stochastic transitional probability matrix $\boldsymbol{P}$ shows the transition rates in per hour of a continuous Markov process.
(a) Construct the state space diagram and discuss particular features of it.
(b) Evaluate the MTTF given that the system starts in state 1.
(c) Derive the differential equations for the system.

$$\boldsymbol{P} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.90 & 0.05 & 0.05 \\ 0 & 0.95 & 0.05 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

3 A pumping station has two identical pumps connected in parallel, each capable of pumping 3000 gallons/hr. If the failure rate and repair rate of each is 0.5 f/hr and 4 r/hr respectively, construct the state space diagram and stochastic transitional probability matrix. Hence evaluate the average hourly throughput of the pumping station. What average throughputs would be obtained if the station had one pump having a capacity of 6000 gallons/hr or three pumps each having a capacity of 2000 gallons/hr?

4 A mission oriented system consists of 3 non-identical, non-repairable components, two of which must operate for system success. If the failure rates are 0.01, 0.05 and 0.1 f/10^6 hr, construct the stochastic transitional probability matrix and hence evaluate the MTTF of the system.

5 A system has one normally operating component A, two standby components B and C and a perfect and instantaneous changeover device. Construct the state space diagram and stochastic transitional probability matrix of the system if all components are repairable, component A is always used as the operating component when available, component B is always used in preference to component C and components do not fail when in the standby mode.

6 A repairable system consists of three non-identical components A, B and C, all of which must work for system success. When one component fails, no further component failures can occur. Construct the relevant state space diagram and hence evaluate general expressions for the individual limiting state probabilities in terms of component failure and repair rates. Evaluate the unavailability of the system in hr/yr if the three components have the following reliability data:

Component	*Failure rate, f/yr*	*Repair rate, r/yr*
A	1.0	365
B	0.1	12
C	0.5	52

7 A continuously operated/repairable system consists of four identical components each having an MTTF of 5000 hr and an MTTR of 25 hr. At least three of the components must work for system success. Construct a state space diagram of the system and calculate the MTTF of the system assuming it starts in the state in which all components are operating. Compare this value with that if all components must work for system success.

8 A base load generating station consists of two 60-MW turbo-alternators each of which is fed by two 45-MW boilers. The average failure rate and average repair time of each boiler is 3 f/yr and 48 hr, respectively, and the average failure rate and average repair time of each turbo-alternator is 2 f/yr and 72 hr, respectively. Evaluate the expected MW output of the complete station.

9 A control system consists of two identical relays operated by one sensor. The operating conditions are as follows:

(i) The system operates successfully if at least one relay is functioning correctly and if the sensor is functioning correctly.

(ii) If the sensor has failed, it can be assumed that the failure rate of both relays is negligible and such failures can be ignored.

(iii) If both relays have failed, it can be assumed that the failure rate of the sensor is negligible and this failure can be ignored.

(iv) There are no restrictions on the number of components that can be undergoing repair at the same time.

(v) The times to failure and times to repair are exponentially distributed.

(vi) The failure rates of the sensor and each relay are 2 f/yr and 4 f/yr, respectively.

(vii) The repair times of the sensor and each relay are 20 hr and 30 hr, respectively.

Draw the state space diagram for this system and construct the stochastic transitional probability matrix. Calculate the mean time to failure of the system if it is started when all three components are functioning correctly.