Problems

1 A man's exercise habits are as follows. If he does his exercise one day, he is 70% sure not to do them the next day. On the other hand, if he does not do his exercises one day he is 60% sure not to do them the next day. In the long run, how often does he do his exercises?

2 A tax consultant has a contract for three cities: X, Y and Z. He never stays in any city more than one day. If he visits city X, then the next day he visits city Y. If, however, he visits either Y or Z, then the next day he is twice as likely to visit city X as the other city. In the long run, how often does he visit each city?

3 A discrete process has the state diagram shown in Figure 8.5. Find the probabilities of being in each of the three states after three steps using a tree diagram given that the process started in State 1. Determine the limiting state probabilities and the mean number of steps to enter State 3 if the process starts each time in State 2.

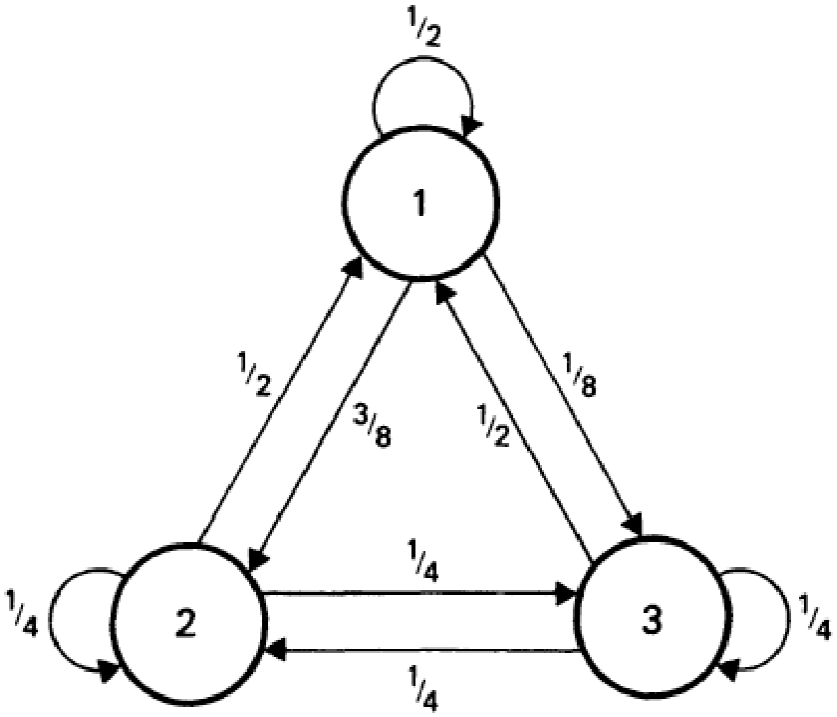

Fig. 8.5

4 A player has 3 dollars. At each play of a game, he loses a dollar with a probability of 3/4 but wins 2 dollars with probability of 1/4. He stops playing if he has lost his 3 dollars or he has won at least 3 dollars. Find the transient probability matrix of the Markov chain. What is the probability that there are at least 4 plays of the game?

5 A gambler's luck follows a pattern: If he wins a game, the probability of winning the next game is 0.8. However if he loses a game, the probability of losing the next game is 0.7. There is an even chance that he wins the first game.
   (a) What is the probability that he wins the second game?
   (b) What is the probability that he wins the third game?
   (c) In the long run how often does he win?

6 Each year a man trades his car for a new car. If he has a Chrysler he trades for a Plymouth. If he has a Plymouth he trades it for a Ford. However, if he has a Ford, he is just as likely to trade it in for a Chrysler or a Plymouth. In 1977 he bought his first car which was a Ford.
   (a) Find the probability that he has a
      (i) 1979 Ford
      (ii) 1979 Chrysler
      (iii) 1980 Plymouth.
   (b) In the long run how often will he have a Ford?

7 A psychologist makes the following assumptions concerning the behaviour of mice subjected to a particular feeding schedule. For any particulat trial 80% of the mice that went right on the previous experiment will go right on this trial and 60% of those mice that went left on the previous experiment will go right on this trial. If 50% went right on the first trial, what would he predict for
   (a) the second trial
   (b) third trial
   (c) the thousandth trial.

8 There are 2 white marbles in urn A and 3 red marbles in urn B. At each step of the process a marble is selected from each urn and the two marbles selected are interchanged. If the state a_i designates the number of i red marbles in urn A,
   (a) Find the transitional probability matrix of the process.
   (b) What is the probability that there are 2 red marbles in urn A after 3 steps?
   (c) In the long run what is the probability that there are 2 red marbles in urn A?

9 Two boys, b_1, b_2 and girls g_1, g_2 are throwing a ball from one to another. Each boy throws the ball to the other boy with a probability 0.5 and to each girl with a probability 0.25. On the other hand, each girl throws the ball to a boy with probability 0.5 and never to the other girl. In the long run how often does each receive the ball?

10 A man's smoking habits are as follows. If he smokes filter cigarettes this week, he switches to non-filter ones the next week with a probability of 0.2. On the other hand if he smokes non-filter ones this week there is 0.7 probability that he will smoke non-filter cigarettes the next week. In the long run how often does he smoke filter cigarettes?

11 A system can reside in one of the three mutually exclusive states shown in the state space diagram of Figure 8.6. The values shown are the probabilities of making the related transition at the end of each discrete time interval of 1 hr. States 1 and 2 represent system success and state 3 represents system

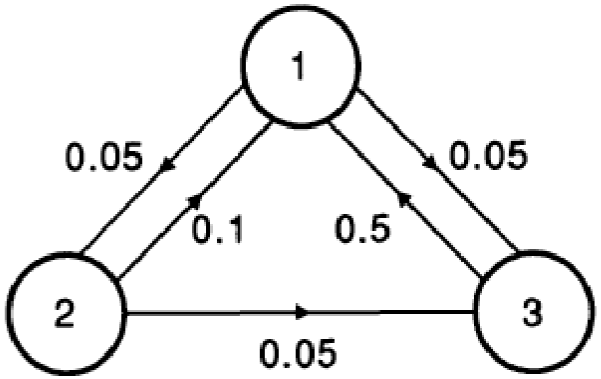

Fig. 8.6

failure. The system starts in state 1. Calculate:

(a) the probability of residing in each state after three time intervals;

(b) the limiting state availability of the system.