Problems

1 A system consists of three black boxes A, B and C. These may be arranged in any one of the four configurations shown in Figure 7.6. The individual

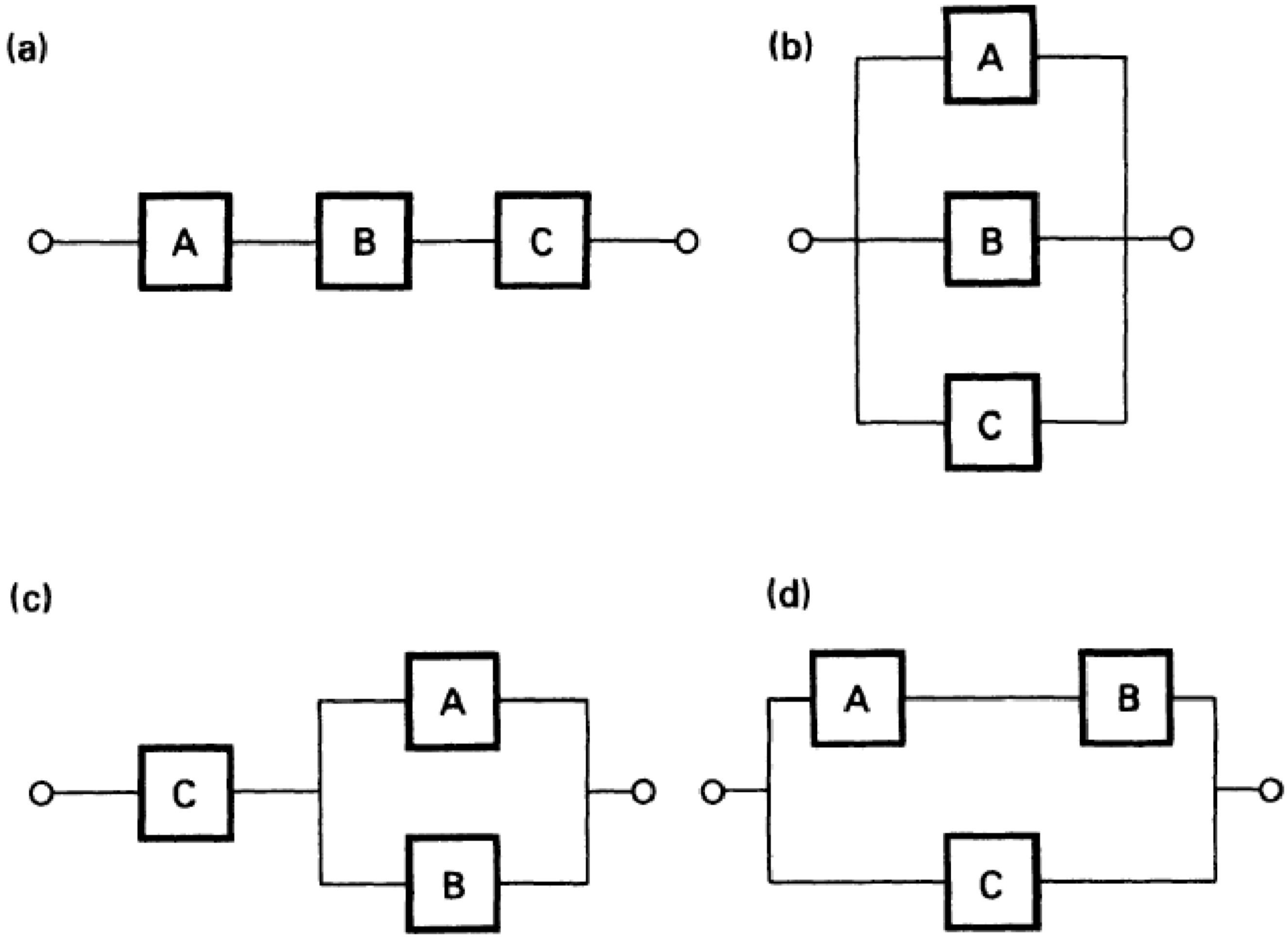

Fig. 7.6

component reliabilities are

$$R_A(t) = e^{-\alpha t}$$

$$R_B(t) = e^{-\beta t}$$

$$R_C(t) = e^{-\gamma t}$$

Write an expression for the system reliability in each of the four cases.

2 The system shown in Figure 7.7 has the following assumed average failure rates

	f/1000 hr
Signal Supply	0.010
Transmission Link 1	0.015
Transmission Link 2	0.027
Receiver	0.011

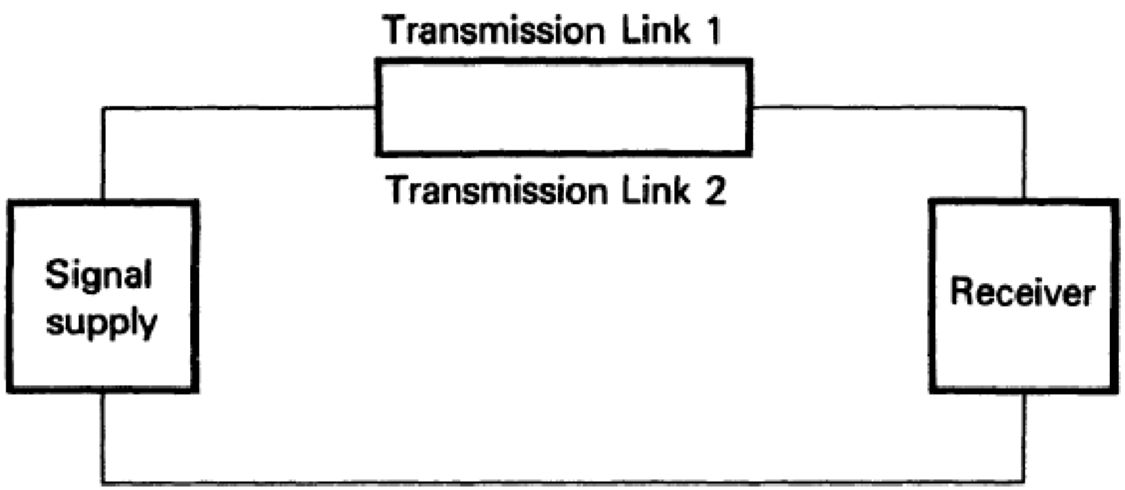

Fig. 7.7

This circuit is left operating for a period of 1000 hr. What is the probability of it still operating at the end of this time? If transmission link 2 is removed from service, what is the probability of the system operating at the end of a 1000 hr period?

3 A system contains a series string of components for which $\sum \lambda = 0.005$ f/hr. What is the reliability of this system for a mission time of 10 hours? If a similar system is placed in parallel what is the reliability of the configuration for a 10 hr mission?

4 For the stand-by system shown in Figure 7.8, mission success requires at least

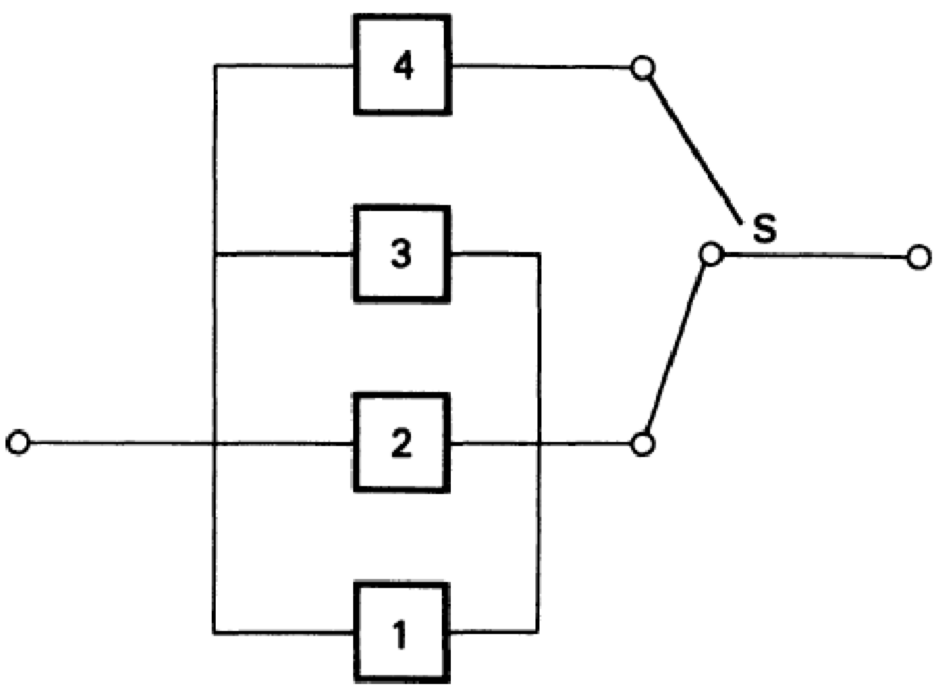

Fig. 7.8

two components. Components 1, 2 and 3 are in parallel and component 4 is in stand-by. Assuming 100% reliable sensing and changeover arrangement, develop the expression for the system reliability. Assume constant failure rates, λ_1, λ_2, λ_3 and λ_4 for components 1, 2, 3 and 4 respectively.

5 Calculate the reliability of the system shown in Figure 7.9 for a 100 hr

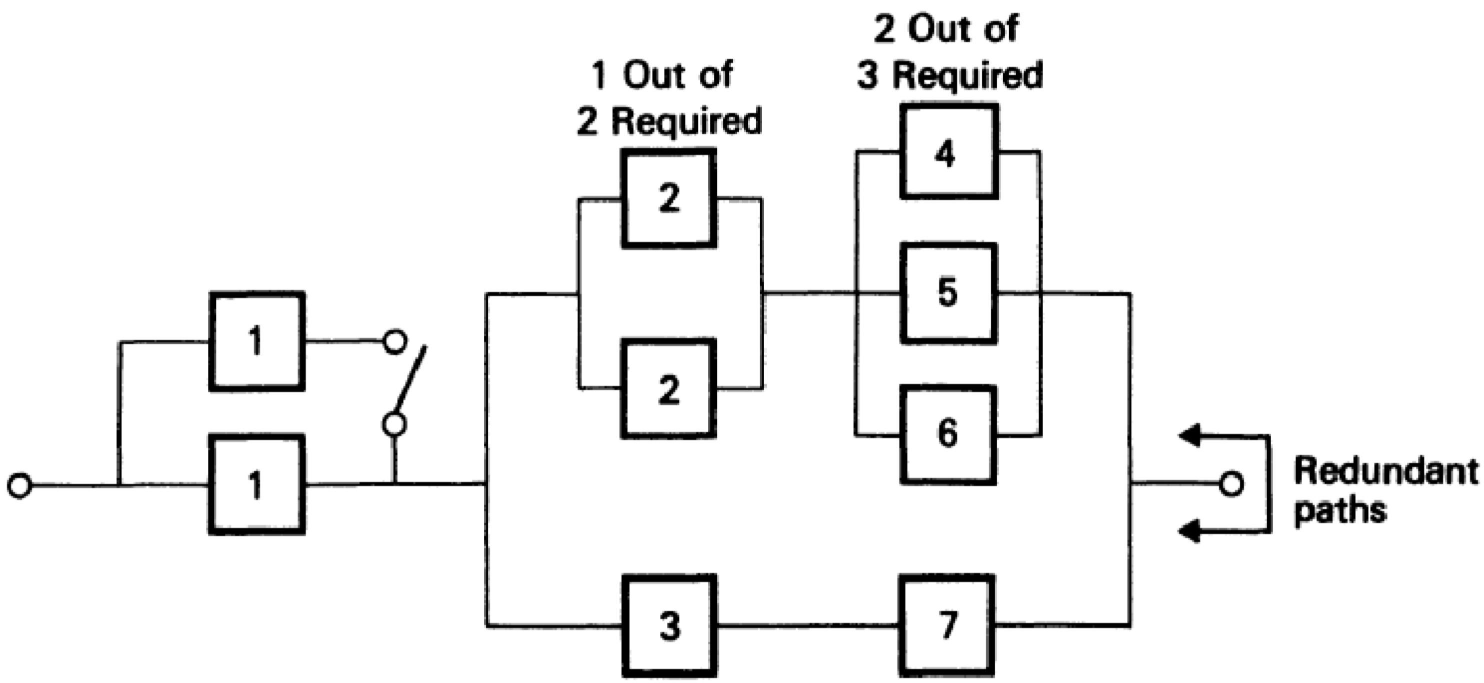

Fig. 7.9

mission.

$\lambda_1 = 12 \times 10^{-5}$ f/hr

$\lambda_2 = 2 \times 10^{-4}$ f/hr

$\lambda_3 = 1 \times 10^{-5}$ f/hr

$\lambda_4 = \lambda_5 = 10 \times 10^{-5}$ f/hr

$\lambda_6 = 5 \times 10^{-5}$ f/hr

$\lambda_7 = 10 \times 10^{-5}$ f/hr

6 (a) Calculate the reliability of the system shown in Figure 7.10 for a

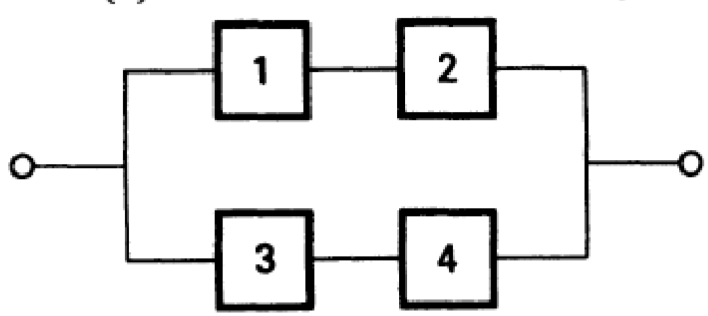

Fig. 7.10

1000 hr mission.

$\lambda_1 = 1 \times 10^{-5}$ f/hr; $\lambda_2 = 10 \times 10^{-5}$ f/hr;

$\lambda_3 = 2 \times 10^{-4}$ f/hr; $\lambda_4 = 5 \times 10^{-5}$ f/hr.

(b) What is the mean time to failure for this system.
(c) Repeat part (a) but with the following additional data.

Component	*1*	*2*	*3*	*4*
Mean wearout time, hr	800	1600	5000	6000
Standard deviation, hr	400	600	200	1000

The mission starts at $T = 1000$ hr in the life of the system.

7 Develop an equation for the reliability of the system shown in Figure 7.11.

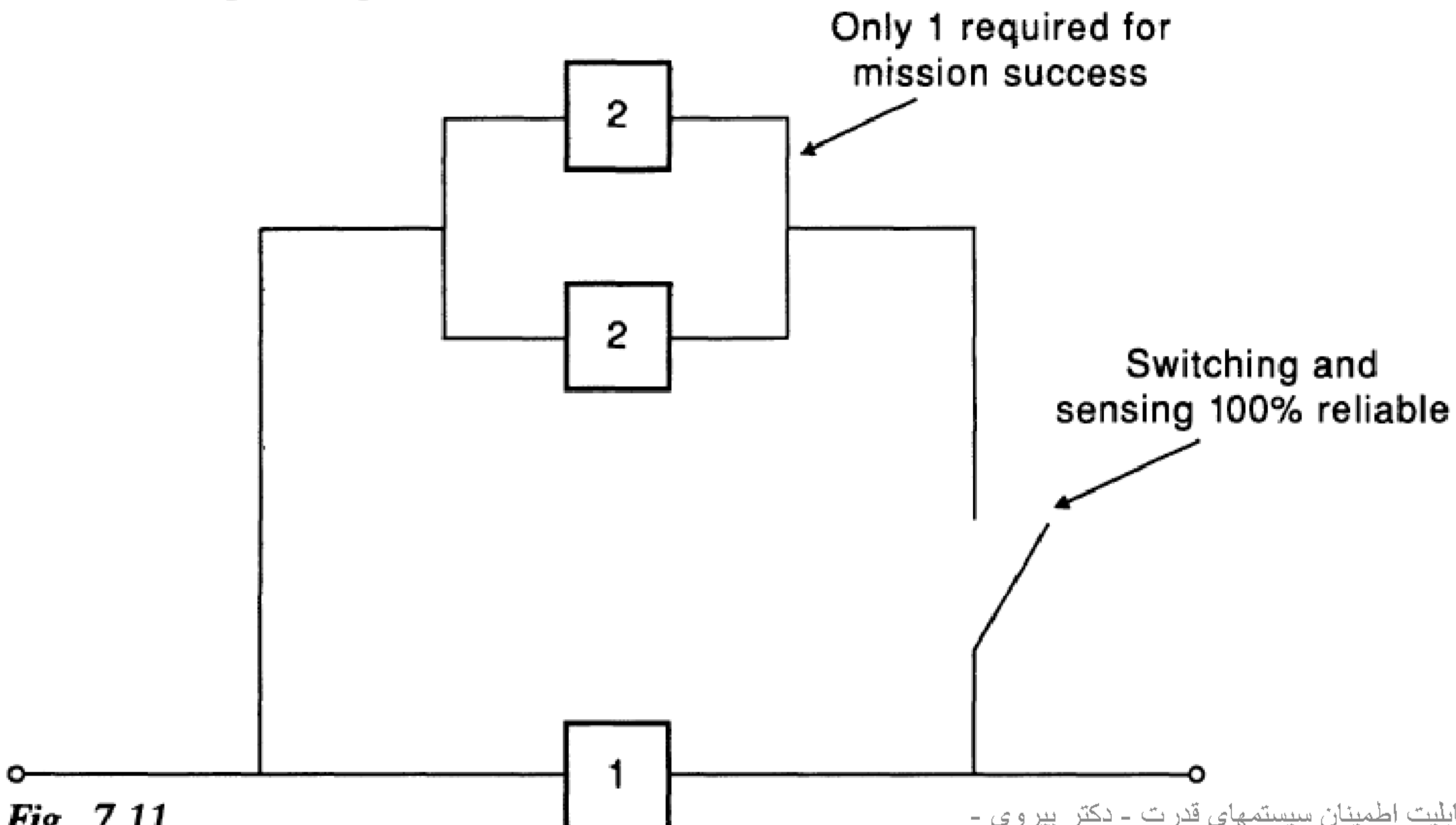

Fig. 7.11

Use your equation to calculate the system reliability for a mission of 1000 hours if $\lambda_1 = 5 \times 10^{-4}$ f/hr and $\lambda_2 = 10 \times 10^{-4}$ f/hr.

8 Calculate the reliability of the system shown in Figure 7.12 for a mission time of 500 hours. The component failure rates are

$\lambda_1 = 2 \times 10^{-4}$ f/hr, $\lambda_2 = 3 \times 10^{-4}$ f/hr, $\lambda_3 = 1 \times 10^{-3}$ f/hr,

$\lambda_4 = 4 \times 10^{-4}$ f/hr, $\lambda_5 = 2.5 \times 10^{-4}$ f/hr, $\lambda_6 = 2.5 \times 10^{-4}$ f/hr,

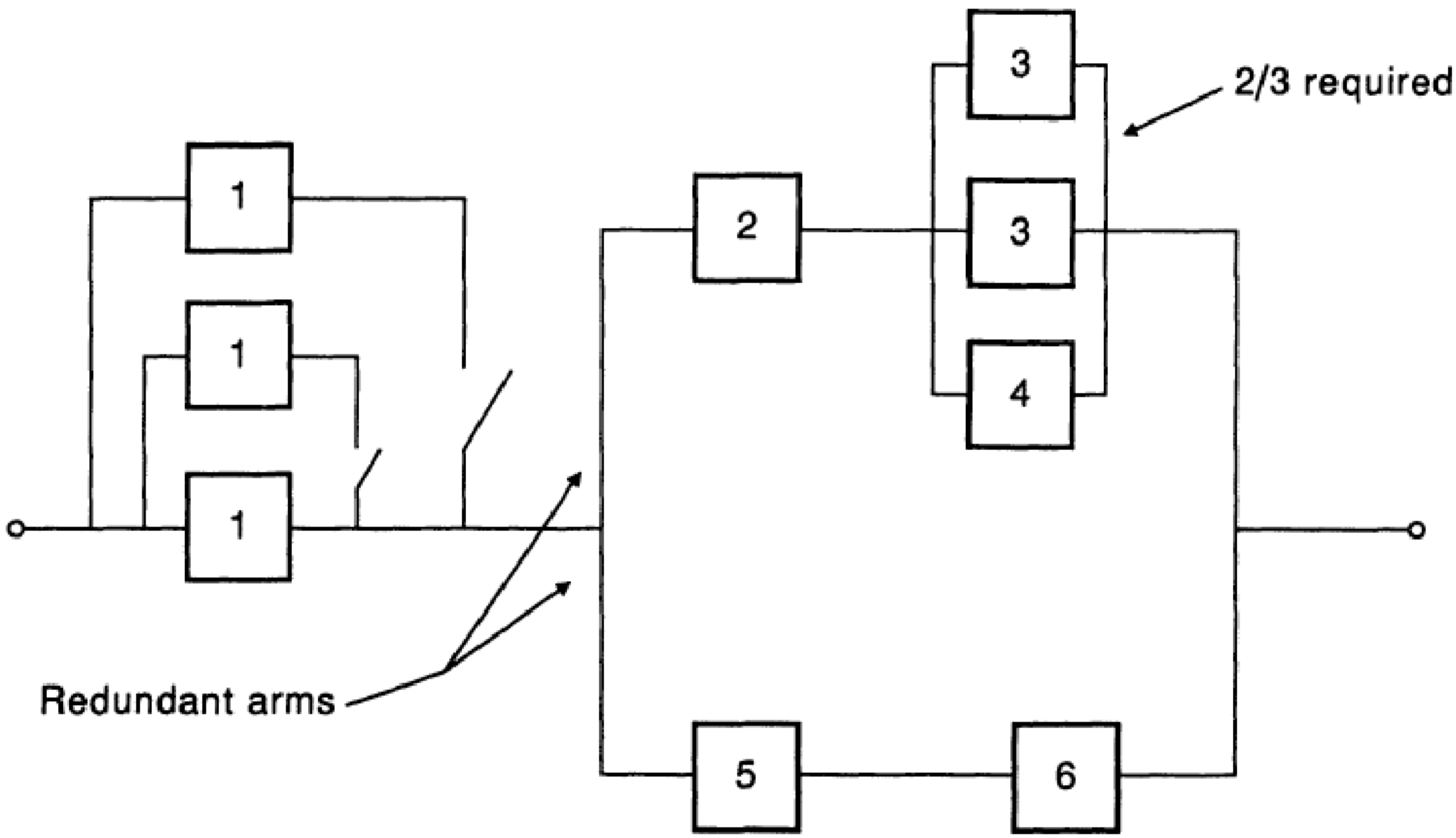

Fig. 7.12

9 Consider the system shown in Figure 7.13. Assume that the signal can flow only in the directions shown.

(a) Develop an equation for the reliability of this system.

(b) Use your equation to calculate the system reliability if all components have a reliability of 0.9.

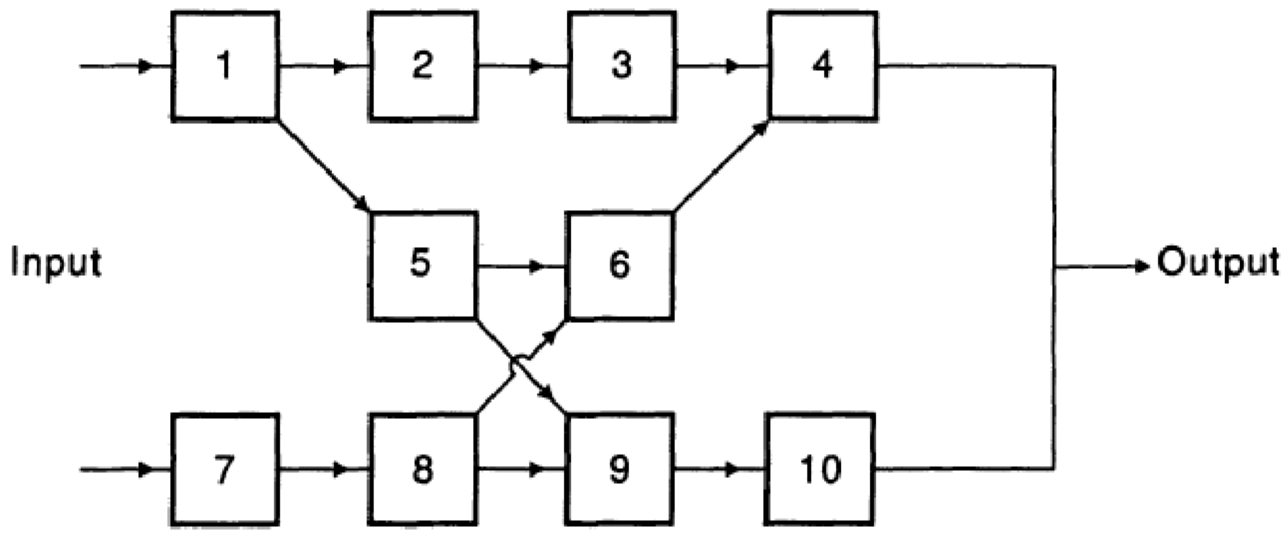

Fig. 7.13

(c) Plot the system mission reliability as a function of time if

$$\lambda_1 = \lambda_3 = \lambda_5 = \lambda_7 = \lambda_9 = 0.001 \text{ f/hr}$$

$$\lambda_2 = \lambda_4 = \lambda_6 = \lambda_8 = \lambda_{10} = 0.002 \text{ f/hr}.$$

(d) Repeat (c) for the case in which the failure rates are doubled.

(e) What are the mean times to failure for this system using the data in (c) and (d)? Obtain an approximate solution using a generalised numerical analysis routine.

10 A system is composed of three elements. Component 1 is in series with the parallel redundant combination of components 2 and 3, which are identical.

$$\lambda_1 = 1\ \text{f/yr}, \qquad \lambda_2 = \lambda_3 = 3\ \text{f/yr}$$

$$\mu_1 = 98\ \text{r/yr}, \qquad \mu_2 = \mu_3 = 50\ \text{r/yr}$$

(a) Calculate the reliability of this system for a mission of 1000 hr.

(b) What is the mean time to failure for this system?

11 Show that, for a component operating in its useful life period, the reliability of the component over any given time period is independent of the prior operating time. For the system shown in Figure 7.14, successful operation requires that at least one path between input and output is good. Show that the reliability of the system R_s is given by

$$R_s = 4R_1R_2 - 2R_1^2R_2 - 4R_1^2R_2^2 + 4R_1^3R_2^2 - R_1^4R_2^2$$

where R_1 is the reliability of components 1, 2, 3, 4 and R_2 is the reliability of components 5, 6. If all the components are in their useful life period and the mean time to failure of the components with reliability R_1 is 2000 hr and of those with reliability R_2 is 4000 hr, what is the reliability of the system for the first 1000 hr of operation? What is the mean time to failure of the system?

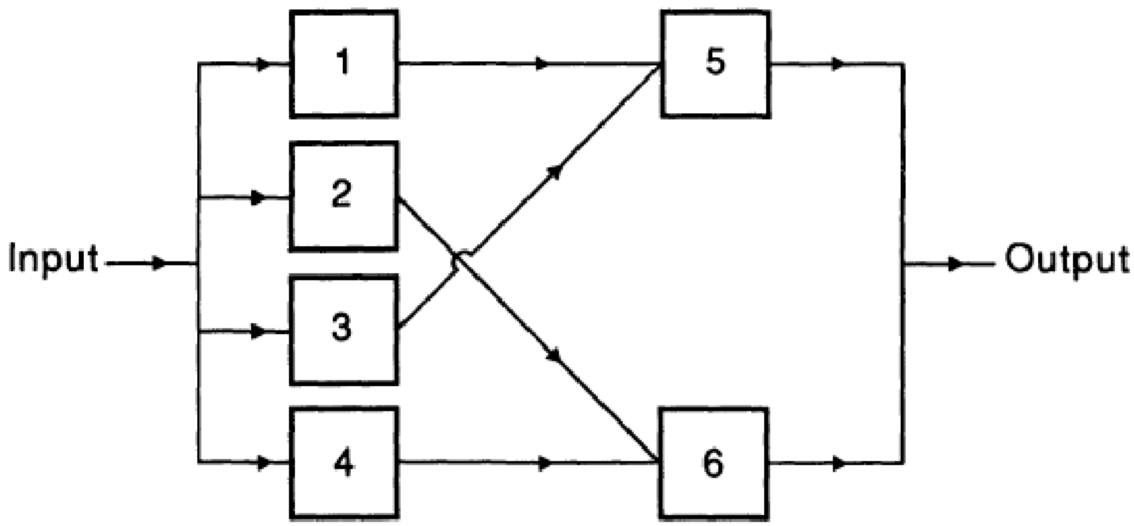

Fig. 7.14

12 A mission oriented (non-repairable) system consists of three identical sensors, two identical relays and two identical control switches. For the system to operate successfully, two of the sensors must operate, one of the relays must operate, and both of the switches must operate. The following conditions apply:

(i) The times to failure of all components are exponentially distributed.

(ii) None of the components can be repaired or replaced during the mission time.

(iii) The MTTF of each sensor is 5 years, of each relay is 2.5 years, and of each switch is 10 years.

Derive a general expression for the time dependent value of system reliability and hence evaluate:

(a) the probability of system failure for a mission time of 1000 hr,

(b) the MTTF of the complete system.

13 A control system consists of three separate, identical and statistically independent channels, at least two of which must operate for system success. Each channel consists of two identical sensors and one actuator. At least one of the sensors and the actuator must operate for success of a channel. All components operate in their useful life period, the MTTF of each sensor is 5000 hr and the MTTF of each actuator is 6000 hr. Evaluate:

(a) the MTTF of each channel;

(b) the probability of the system surviving the first 500 hr without failure.

14 A control system consists of three main components: a transducer, an amplifier and an actuator, having failure rates of 0.35, 0.05 and 0.02 f/yr, respectively. All components operate in their useful life period.

(a) Evaluate the probability of this system surviving for 1 year without failure and its MTTF.

(b) What is the minimum number of transducers that must be connected in parallel redundancy to increase the probability of the system surviving for 1 year without failure to greater than 0.9?

(c) Evaluate the new probability of surviving for 1 year and the MTTF after the improvement in (b) has been done.