Problems

1 A manufacturer produces 10 items of a special product each year. If the items are not sold within the year they must be discarded. Past experience has indicated that the demand for the product is a Poisson distributed random variable with an expectation of 8. If a profit of \$7.00 is made on every item which is sold and a loss of \$3.00 results from having to discard an item, calculate the expected profit realized by the supplier on the 10 items that he produces.

2 A highway has a travel rate of 120 vehicles per hour. What is the probability of 0, 1, 2, 3, 4 vehicles passing a certain point during a given 30 second interval?

3 The number of oil tankers arriving at a refinery each day has a Poisson distribution with parameter $np = 2$. Present port facilities can service three tankers arriving in any one day. Tankers in excess of three must be sent to another port.

(a) On any given day, what is the probability of having to send tankers away?

(b) How much must present facilities be increased to permit handling all tankers on 90% of the days?

(c) What is the expected number of tankers arriving per day?

(d) What is the most probable number of tankers arriving daily?

(e) What is the expected number of tankers serviced daily?

(f) What is the expected number of tankers turned away daily?

(g) Repeat (a) to (f) if the port facilities can service 2 tankers in any one day.

(h) Repeat (a) to (f) if the port facilities can service 4 tankers in any one day.

4 A communication system has on the average 26 component failures per year of the same plug-in element. These elements are replaced from available stock which is replenished every two weeks. How large should the stock be to prevent with a 90% (or greater) probability, the communication system being forced out of service. What should the stock be if the average number of component failures per year is 13?

5 The ABC Auto Supply depot orders stock at the middle of the month and receives the goods at the first of the next month. The average number of requests for fuel pump XY33 is 4 per month. If on April 15, 2 of these fuel pumps are in stock and an additional 5 are ordered (to be received by May 1), what is the probability that the ABC Depot will not be able to supply all the requests for XY33 in the month of May? The demand is completely random. (Requests for pumps are not carried over from one month to the next.)

6 If the diameters of ball bearings are normally distributed with mean 0.6140 inches and standard deviation 0.0025 inches determine the percentage of ball bearings with diameters.
   (a) Between 0.610 and 0.618 inches.
   (b) Greater than 0.617 inches.
   (c) Less than 0.608 inches.

7 The average mark on a final exam is 70 and the standard deviation is 10. The top 10% of the class will receive A's. What is the minimum mark a student must get to receive an A?

8 A machine produces bolts that are 8% defective. Find the probability that in a random sample of 500 bolts produced by the machine,
   (a) at most 50,
   (b) between 30 and 50,
   (c) between 35 and 45,
   (d) 55 or more of the bolts will be defective.
   (Use the normal distribution as an approximation to the binomial).

9 The life of a bearing is normally distributed with an average value of 200 hours and a standard deviation of 100 hours. What is the probability that the bearing fails sometime between 1964 hrs and 2016 hrs?

10 A component operating during its useful life has a reliability of 90% for a mission of 50 hours. What would the component reliability be for a mission of 100 hours?

11 The variations in the output power of a motor are found to follow a gamma distribution with parameters $\beta = 3$ and $\alpha = 100\,\text{kW}$. What is
   (a) the probability that the power output is less than 200 kW?
   (b) the value of this probability if it is approximated to a normal distribution and then to a lognormal distribution?

12 A large number of identical relays have times to first failure that follow a Weibull distribution with parameters $\beta = 0.5$ and $\alpha = 10$ years. What is the probability that a relay will survive (a) 1 year, (b) 5 years, and (c) 10 years without failure and what is the value of MTTF.

13 What are the conditions for the function shown in Figure 6.29 to be a probability density function. Under these conditions evaluate:
   (a) the cumulative failure distribution
   (b) survivor function
   (c) hazard rate
   (d) expected value
   (e) standard deviation
   (f) probability that $\left(a - \frac{b}{2} < t \leq a + \frac{b}{2}\right)$.

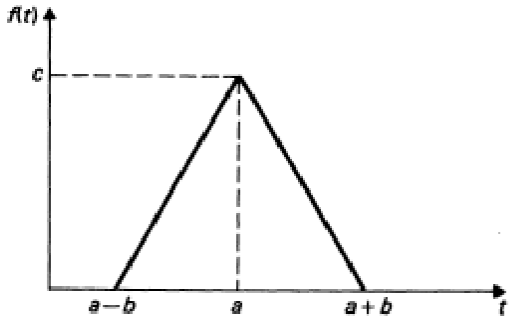

Fig. 6.29

14 The hazard rate of a device is $\lambda(t) = 1/\sqrt{t}$. Deduce
(a) the probability density function,
(b) the survivor function,
(c) the expected value, and
(d) the variance.

15 A person has developed the following procedure for getting to work in the morning. If her breakfast takes less than 10 minutes, she takes route A, which has a mean travel time of 14 minutes and a standard deviation of 3 minutes. If it takes more than 10 minutes, she takes route B, which has a mean travel time of 11 minutes, and a standard deviation of 6 minutes. Travel times and breakfast duration are normally distributed, the latter with a mean time of 8 minutes and a standard deviation of 3.5 minutes.
(a) What is the expected travel time?
(b) What is the probability of a travel time over 13 minutes?
(c) Given that the travel time was less than 13 minutes, what is the probability that her breakfast took more than 10 minutes?

16 An electricity supply company has 2000 km of a specific type of cable operating in their system. The total number of failures recorded in each year of a 10-year period were 36, 37, 44, 42, 44, 38, 39, 41, 38, 41. Determine the expected value and standard deviation of the failure rate for this type of cable. A particular load point is supplied by a cable of this type. The cable length is 10 km. Calculate for a 20-year period:
(a) the expected number of failures of this 10-km cable;
(b) the most probable number of failures;
(c) the probability that there will be seven or more failures
State all the assumptions you have made in these calculations.