Problems

1 The probability that a man will be alive in 10 years is 0.8 and the probability that his wife will be alive in 10 years is 0.9. Find the probability that in 10 years:
   (i) Both will be alive;
   (ii) Only the man will be alive;
   (iii) Only the wife will be alive;
   (iv) At least one will be alive.

2 The probability of Inspector #1 on a production line finding a defective item is 0.8 and of Inspector #2, down the line, 0.7. What is the probability of a defective item getting through?

3 Three urns contain, respectively, 1 white and 2 black balls, 2 white and 1 black balls, 2 white and 2 black balls. A blindfold man transfers one ball from the first urn into the second, then one ball from the second urn into the third. A ball is drawn from the third urn. What is the probability of its being white?

4 A worker operates three machines. The probability that for the duration of an hour a machine does not require the attention of the worker is 0.9 for the first machine, 0.8 for the second and 0.85 for the third. What is the probability that in any one hour none of the machines require his attention? What is the probability that at least one of the machines does not require any attention during any one hour?

5 Two dice are tossed together. Let A be the event that the sum of the faces are odd, B the event that at least one is a one. What is the probability that:
   (a) Both A and B occur;
   (b) Either A or B or both occur;
   (c) A and not B occurs;
   (d) B and not A occurs?

6 A box contains a normal coin and a two headed coin. A coin is selected at random and tossed. If heads appear, the other coin is tested, if tails appear, the same coin is tossed.
   (a) Find the probability that heads appear on the second toss.
   (b) If heads appeared on the second toss, find the probability that it also appeared on the first toss.

7 A shopper buys two cartons of a dozen eggs each. His habit is to inspect 3 eggs picked at random from each carton and to reject the carton if he finds one or more cracked eggs. If the first carton contains two cracked eggs and the second, one cracked egg, find the probability that:
   (a) Carton 1 is rejected and carton 2 is accepted.
   (b) Both cartons are accepted.
   (c) Neither carton is accepted.

8 A piece of equipment contains six identical items and it is known that three of them are defective. The items are tested one after the other until the three defective items are found.
   (a) What is the probability that the testing process is stopped on the (i) third test (ii) fourth test.
   (b) If the process is stopped on the fourth test, what is the probability that the first item is not defective.

9 Three persons work independently at solving a given design problem. The respective probabilities that they solve it are 1/4, 1/3, 1/2. What is the probability that the problem will be solved?

10 If the probability of a child being male is 0.55, what is the probability of having three daughters in succession?

11 The probability of A hitting a target is 1/2 and the probability that B hits it is 1/4. If each fire once, what is the probability that the target is hit:
(a) twice;
(b) only by A;
(c) only by B;
(d) not at all;
(e) if A can fire only once, how many times must B fire so that there is at least 90% probability that the target will be hit?

12 A point is selected at random inside a square whose side length is 4. What is the probability that its distance to any corner is greater than 2?

13 A statistics student plays a dice game with his lab instructor. If the student rolls a prime number with one roll of a die, he wins that number of dollars but if a nonprime number occurs he loses that amount. What is the student's mathematical expectation? Should he be playing the game? Repeat the problem if two dice are used.

14 A pair of fair dice is tossed. Find the probability that at least one of the two numbers is greater than 4.

15 A bushpilot faces the following situation. His single-engined aircraft is on fire. He could attempt to crash land on the beach ahead, in which case he will need assistance or he could attempt to reach an emergency strip at which he can land safely and assistance will be available. Using the estimates given below what should he do if his objective is to stay alive?
(a) odds on reaching emergency strip versus mid-air explosion 1:4;
(b) probability of surviving mid-air explosion 0;
(c) probability of reaching beach 0.8;
(d) probability of surviving beach landing 0.5;
(e) probability of receiving necessary assistance after a beach landing
—by air rescue 0.3,
—by passing boat 0.2.

16 The two digits are selected at random from the digits 1 through 9. If the sum of the two digits is even, find the probability that both numbers are odd.

17 Two machines produce the total output of a factory. Machine 1 produces 70% and machine 2 produces 30% of the output. Five percent of the output of machine 1 is defective and 8% from machine 2. If a finished item is selected at random, what is the probability of it being defective?

18 A box contains a fair coin and a two-headed coin. A coin is selected at random and tossed. If a head appears, the other coin is tossed; if a tail appears the same coin is tossed.
(a) find the probability that a head appears on the second toss;
(b) if a head appears on the second toss, find the probability that it also appeared on the first toss.

19 A coin is tossed. If the outcome is a head, a company hires an engineer. If the outcome is a tail, the company hires two technicians. After two tosses of

the coin, one man is fired at random. What is the probability that he is an engineer?

20 A player tosses two fair coins. He wins \$1.00 if one head appears and \$2.00 if two heads appear. On the other hand he loses \$5.00 if no heads appear. What is his expected gain in this game? Should he be playing this game?

21 Two men, A and B, play in a chess tournament. A has a slight edge over B with odds of 11 to 9. They keep playing until one of the two wins three games in a row. What is the probability that they play more than four games?

22 A bag contains an assortment of green and red marbles. Student A knows the contents and after informing student B that the bag contains only red and green marbles invites him to put his hand in the bag and take one. Student B pulls out a green marble and on a second try pulls out another green marble. He comments that the bag seems to be a little short on red marbles and asks what are the odds against getting a third green if he tries again. Student A thinks for a moment and then replies "exactly twice what they were against getting the second green". How many green and red marbles were there in the bag at the start?

23 Three boxes containing red, white and blue balls are used in an experiment. Box 1 contains three red, three white and two blue balls. Box 2 contains one red, one white and one blue ball. Box 3 contains three red, one white and five blue balls. The experiment consists of drawing a ball at random from box 1 and replacing it in box 2, then drawing a ball at random from box 2 and placing it in box 3.

(a) Draw the probability distributions of the number of red, white and blue balls in box 3.

(b) What are the expected number of red, white and blue balls in box 3?

(c) What are the variances of the probability distributions of the number of red, white and blue balls in box 3?

(d) Given that at the end of the experiment, there were six blue balls in box 3, what is the probability that a white ball was drawn from box 1?